Tanta University

Department: Engineering Physics and Mathematics
Total Marks: 40 Marks

Faculty of Engineering

Course Title: Numerical Analysis	Course Code: --	Year: Mechatronic Program
Date: May27, 2019(2nd term)	Allowed time: 3 Hr.	No. of Pages: (2)

Solve All The Following Questions

QUESTION NO.(1) {10 MARKS}

i- By using one of the iterative techniques, find the smallest positive root for the following nonlinear algebraic equations correct to six decimal places, $f(x) = \cos x \cosh x - 1$, starting with the initial guess $x_o = \frac{3\pi}{2}$.

ii- Set up the Lagrange's interpolation polynomial and Newton's divided difference formula to determine the polynomial of degree ≤ 5 that interpolate the following data,

x	1	2	3	4	5	6
f(x)	14.5	19.5	30.5	53.5	94.5	159.5

By using the resulting polynomial, estimate the value of $f(4.5)$ and compare your result with the exact value $f(4.5) = 71.375$.

QUESTION NO.(2) {10 MARKS}

Evaluate the following Bessel's function integral formula

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x \sin\theta)d\theta,$$

at $n = 1$ and $x = 3$ taking the step size $h = \frac{\pi}{8}$, using the following integration rules

i- trapezium rule.

ii- Simpson's 1/3 rule.

Compare your result with the exact value $J_1(3) = 0.3391$.

QUESTION NO.(3) {10 MARKS}

Solve the boundary value problem

$$y'' + \frac{2}{x}y' + \frac{2}{x^2}y = \frac{10}{x^2}\cos(Ln\, x), \qquad y(1) = 1, \qquad y(3) = -1.$$

Over $x \in [1,3]$ with $h = 0.5$, by using central difference approximation to y'' and

i- central difference approximation to y'.

ii- backward difference approximation to y'.

If the exact solution is $y(x) = \frac{1}{x^2}[4.335 - 0.3359\,x^3 - 3x^2\cos(Ln\, x) + x^2\sin(Ln\, x)]$, compare the magnitudes of errors at the nodal points in the two methods.

QUESTION NO.(4) {10 MARKS}

Compute approximations to $x(0.25)$ and $y(0.25)$ for the following system

$$x' = (1 + sin\ t)x + 0.1\,y, \qquad x(0) = 1,$$

$$y' = x + 0.25\,y, \qquad y(0) = 1,$$

over $t \in [0,\ 0.25]$ with time step length h = 0.25 using :

i- Runge-Kutta method of second order (Heun's method) .
ii- Runge-Kutta method of third order.
iii- Runge-Kutta method of fourth order.

****

All the best
Course Examination Committee and Course Coordinators
Dr.Eng. : Mohamed El-Borhamy and the committee